Analytic Theory of Interest Rates
Part I: Solitary Bonds

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An analytic theory of interest rates, carefully constructed starting from first principles, is presented. It aims to provide a conceptual framework to unify the diverse phenomena associated with bonds. This paper contains Part I concerning solitary bonds; the sequel, to be published, will contain Part II concerning aggregates of bonds.

The paper starts with a description and the definitions of the fundamental quantities characterising the innate behaviour of a bond. It then proposes the effective return rate on intrinsic value and the effective interest rate as measures to evaluate the growth of an investment in a bond. The yield-to-maturity rate based on the discounting of cash-flows, conventionally used for that purpose, is next treated in such a manner as to allow detailed comparison with the effective interest rate. This is achieved through the introduction of dimensionless quantities for the rates, the price and the remainder of the term. Figures of the relations between the dimensionless quantities bring out the differences between the effective interest rate and the yield-to-maturity rate. A (zeroth order) pricing model for bonds, which utilises the effective return rate on intrinsic value and the effective interest rate, completes the paper.

Continuous and periodic compounding, the two interest calculation methods in use theoretically and practically, are juxtaposed throughout. Accordingly, the various rates are defined for continuous as well as periodic compounding. An expression in one or more of the rates may hence appear in several disguises, each rate potentially doubling the number. The rules to convert expressions valid for one interest calculation method into the equivalent expressions valid for the other are formulated early on because they make for neat analyses which circumvent tedious derivations.

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The results are applied to three typical investment strategies, one to deploy zero coupon bonds - which pay interest once only at the term's end - and the other two to deploy common bonds - which pay interest regularly each accounting period - where the bondholder has the choice to reinvest the interest he receives or not. It appears that the conventional yield-to-maturity rate normally does not coincide with the effective interest rate for common bonds. Furthermore, over very long holding periods, the effective interest rate converges on a well-defined limit independent of the bond price whereas the yield-to-maturity rate can assume any positive value dependent on the bond price.

The main result of the paper is in the combination of the dimensionless quantities with the model. The yield curve usually consists of a plot of the yield-to-maturity rate against the remaining term. The proposal is to supplant it by a plot of the dimensionless effective interest rate against the dimensionless remaining term. The newly proposed yield curve predicted by the (zero order) model then is a horizontal straight line likely to vary in the course of time.

International Journal of Theoretical and Applied Finance
Vol. 3, No. 3 (2000) 573-574.