

A VERY SIMPLE METHOD FOR CONVERSION OF HYPERBOLIC INTO RECTANGULAR COORDINATES

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Abstract

A very simple method is presented for converting hyperbolic coordinates, such as Decca Hifix, into rectangular grid coordinates. This method can be applied in a short computer programme for the construction of a calibration chart, and is especially suited for use on computers having a very small memory, such as those used on board survey vessels. The programme uses an efficient iteration procedure. Normally only a few steps, typically 3, are enough to obtain the required accuracy of 0.5 m in the rectangular coordinates.

Introduction

In the course of a marine seismic survey in SW Japan, a programme was needed for converting hyperbolic into rectangular grid coordinates. The computer installed on board the ship, a PDP-8/I, had a small memory capacity of 4K only so that a very simple, compact programme was required. An inspection of the literature showed that a suitable method was not yet available. An extensive review of previous methods can be found in ref. [1]. In a first attack I tried to use an analytical approach as given by Verstelle [2]. The computations in this approach turned out to be still too complicated to be handled by our computer, and I adopted the iteration procedure presented below. The number of steps needed in the iteration is small, usually 3. In the past few months the programme has worked quite satisfactorily.

The mathematical problem and its solution

The mathematical model of radio positioning systems using hyperbolic coordinates, such as Decca Hifix, is very well known [1]. In the following section a short exposé is given to define the symbols.

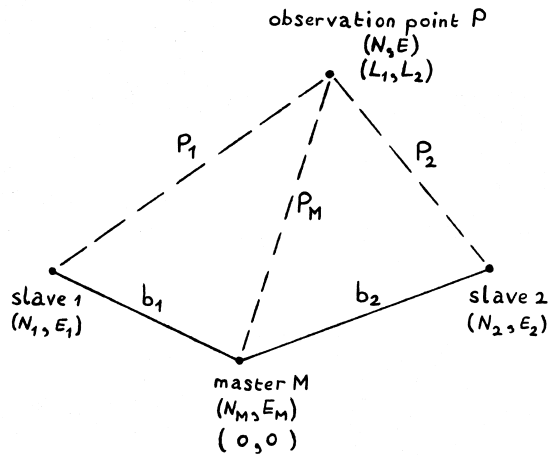


Figure 1. Position of the observation point P relative to the master station M, and the slave stations 1 and 2.

In radio positioning systems the hyperbolic lattice over a part of the earth is set up by three transmitting stations, the master M, slave 1 and slave 2,

as shown in figure 1. All three stations emit a phase-synchronized signal, which may be considered a continuous sine wave. At the observation point P the phase difference between the master and both slaves can be measured. With a known propagation speed and frequency, the phase difference can be converted into a difference in distance from the master and each slave. The distance equivalent to a phase difference of 2π radians is known as a ‘lane’. The origin of the hyperbolic coordinates is chosen at the master.

In the mathematical model chosen a flat earth is assumed. In this approximation the spheroidal hyperbolae become plane hyperbolae, which simplifies the approach. If a suitable chart projection is used (in this case U.T.M.) a scale factor can be introduced to account for the errors due to the projection [2]. So, we arrive at the well-known formulae for the phase difference at a point P, the so-called lane numbers

$$L_1 = \frac{b_1 + P_M - P_1}{\lambda}, \quad (1)$$

$$L_2 = \frac{b_2 + P_M - P_2}{\lambda}, \quad (2)$$

$$\lambda = \frac{v}{f}, \quad (3)$$

where

- L_1 = lane number of the pattern set up by the master and slave 1,
- L_2 = lane number of the pattern set up by the master and slave 2,
- b_1 = distance between the master and slave 1, or baseline 1,
- b_2 = distance between the master and slave 2, or baseline 2,
- P_M = distance from observation point P to the master,
- P_1 = distance from observation point P to slave 1,
- P_2 = distance from observation point P to slave 2,
- λ = wavelength of the transmitted signal,
- v = propagation velocity of the transmitted signal,
- f = frequency of the transmitted signal.

Further,

$$P_1 = \frac{\left\{ (N - N_1)^2 + (E - E_1)^2 \right\}^{1/2}}{S_F}, \quad (4)$$

$$P_2 = \frac{\left\{ (N - N_2)^2 + (E - E_2)^2 \right\}^{1/2}}{S_F}, \quad (5)$$

$$P_M = \frac{\left\{ (N - N_M)^2 + (E - E_M)^2 \right\}^{1/2}}{S_F}, \quad (6)$$

where in the rectangular grid coordinates

$$\begin{aligned} N, E &= \text{the northing and easting of point P,} \\ N_M, E_M &= \text{the northing and easting of the master,} \\ N_1, E_1 &= \text{the northing and easting of slave 1,} \\ N_2, E_2 &= \text{the northing and easting of slave 2.} \end{aligned}$$

The scale factor S_F accounts for the distortion of distances due to the chosen U.T.M. projection system and is a given.

In the computer programme the grid coordinates of point P are calculated from the observed lane numbers by means of an iteration procedure. Firstly, a rough estimation of the northing and easting of point P is obtained somehow. Inserting these into formulae (4) - (6) the lane numbers L_1 and L_2 at point P are calculated with formulae (1) - (3). The differences between the actual, observed lane numbers and the calculated lane numbers are used to obtain a second and better approximation of the northing and easting of point P. This approximation is used for a renewed calculation of the lane numbers L_1 and L_2 , which in turn can be used for a still better approximation of the northing and the easting. These steps may be repeated till a certain accuracy is reached, i.e. till the difference between the last two approximations of northing and easting of point P is less than a certain chosen limit.

As the lane numbers L_1 and L_2 are functions of N and E , we may expand L_1 and L_2 with respect to a point N^1, E^1 into a Taylor series:

$$L_1(N, E) = L_1(N^1, E^1) + \frac{\partial L_1}{\partial N} (N - N^1) + \frac{\partial L_1}{\partial E} (E - E^1) + \dots, \quad (7)$$

$$L_2(N, E) = L_2(N^1, E^1) + \frac{\partial L_2}{\partial N} (N - N^1) + \frac{\partial L_2}{\partial E} (E - E^1) + \dots. \quad (8)$$

Starting from a first estimate N^1, E^1 for the northing and easting of point P we can use the expansions to find a better approximation N^2, E^2 . If L_1^0 and L_2^0 are the observed lane numbers and L_1^1 and L_2^1 the lane numbers calculated from N^1, E^1 , we should require that

$$L_1^0 = L_1^1 + \frac{\partial L_1}{\partial N} (N^2 - N^1) + \frac{\partial L_1}{\partial E} (E^2 - E^1), \quad (9)$$

$$L_2^0 = L_2^1 + \frac{\partial L_2}{\partial N} (N^2 - N^1) + \frac{\partial L_2}{\partial E} (E^2 - E^1), \quad (10)$$

where the first derivatives are taken at N^1, E^1 . It follows that

$$N^2 = N^1 + \left\{ \frac{\partial L_2}{\partial E} (L_1^0 - L_1^1) - \frac{\partial L_1}{\partial E} (L_2^0 - L_2^1) \right\} / \left\{ \frac{\partial L_1}{\partial N} \frac{\partial L_2}{\partial E} - \frac{\partial L_1}{\partial E} \frac{\partial L_2}{\partial N} \right\}, \quad (11)$$

$$E^2 = E^1 + \left\{ \frac{\partial L_2}{\partial N} (L_1^0 - L_1^1) - \frac{\partial L_1}{\partial N} (L_2^0 - L_2^1) \right\} / \left\{ \frac{\partial L_1}{\partial E} \frac{\partial L_2}{\partial N} - \frac{\partial L_1}{\partial N} \frac{\partial L_2}{\partial E} \right\}. \quad (12)$$

The first derivatives follow easily from formulae (1) - (6):

$$\frac{\partial L_1}{\partial N} = \left\{ \frac{N^1 - N_M}{P_M} - \frac{N^1 - N_1}{P_1} \right\} \frac{f}{vS_F^2}, \quad (13)$$

$$\frac{\partial L_1}{\partial E} = \left\{ \frac{E^1 - E_M}{P_M} - \frac{E^1 - E_1}{P_1} \right\} \frac{f}{vS_F^2}, \quad (14)$$

$$\frac{\partial L_2}{\partial N} = \left\{ \frac{N^1 - N_M}{P_M} - \frac{N^1 - N_2}{P_2} \right\} \frac{f}{vS_F^2}, \quad (15)$$

$$\frac{\partial L_2}{\partial E} = \left\{ \frac{E^1 - E_M}{P_M} - \frac{E^1 - E_2}{P_2} \right\} \frac{f}{vS_F^2}. \quad (16)$$

From the first estimation N^1, E^1 we can thus arrive at a closer approximation N^2, E^2 with formulae (11) - (16). If this approximation is not yet accurate enough, we repeat the calculation with N^2, E^2 and obtain a third set of values N^3, E^3 . We can continue until the aim is reached: a sufficiently accurate estimate of the true N and E .

An example of a computer programme

The procedure explained above can be used for several purposes. As an example, a programme is presented for calculation of a calibration chart. A calibration chart is a hyperbolic lattice chart on a large scale, used to determine the existence of any phase error in the pattern. The programme is written in FOCAL, a language especially developed for the PDP-8/I computer by Digital equipment Corporation. A description of FOCAL can be found in ref. [3] and [4]. The symbols used in the previous chapter have had to be changed slightly for the programme.

A flow chart of the programme is shown in figure 2. The programme itself is shown in tables 1 and 2. It is subdivided into four parts. In part 1 all basic constants specific to the Hifix chain are gathered. Obviously, if the programme is adopted for a different Hifix chain, these constants must be altered. In part 2 the data of the calibration chart under construction are asked for by the programme. They consist of the exact chart limits expressed in grid coordinates, a rough estimation of the chart limits in hyperbolic coordinates, the increments of the hyperbolic coordinates, and the scale factor. Part 3 is the actual programme and has a control function. At first the heading of the output is printed, then N and E are set equal to the lower chart limits. Two nested loops are used to calculate all lane intersections between the estimated limits of the lane numbers. If an intersection falls within the chart limits, the result is printed, otherwise the intersection is disregarded. The increments of the hyperbolic coordinates determine the density of the calculated lane intersections. Finally, part 4 as a subroutine contains the iteration procedure explained above. The accuracy of the iteration is chosen at 0.5 m, which is thought to be sufficient [2]. In table 3 a type-out is presented of the symbol table of the programme. The short definition behind each symbol was typed in afterwards and is not contained in the programme.

A few remarks should be made. Firstly, no attempts are made to calculate the minimum and the maximum value of each pattern for the area of the calibration chart. Such a calculation would become too large for our computer. Instead in part 2 the programme asks for a rough estimation, and deletes all lane intersections with a northing and a easting outside the limits of the chart. Through this, the simplicity of the programme is paid for by the greater calculation time. Secondly, all northings are diminished by $3 \cdot 10^6$ m. This means that the grid origin is shifted over this distance. This increases the accuracy of the calculations, since in FOCAL the results are calculated up to six significant digits only. In another language this would not be necessary. To draw the attention of the user to this fact at the end of part 1 a type command is given, and the text "ADD 3,000,000 TO ALL PRINTED NORTHINGS." is printed. Finally, in the beginning of the programme northing and easting are set equal to the lower specified limits of the chart. This is a rather arbitrary choice. In the first calculation these values are used for the approximation. In the course of the rest of the programme the northing and easting that happen to be in the memory are used. These values are always very close to the desired values and so the required number of steps is decreased and thus the calculation time.

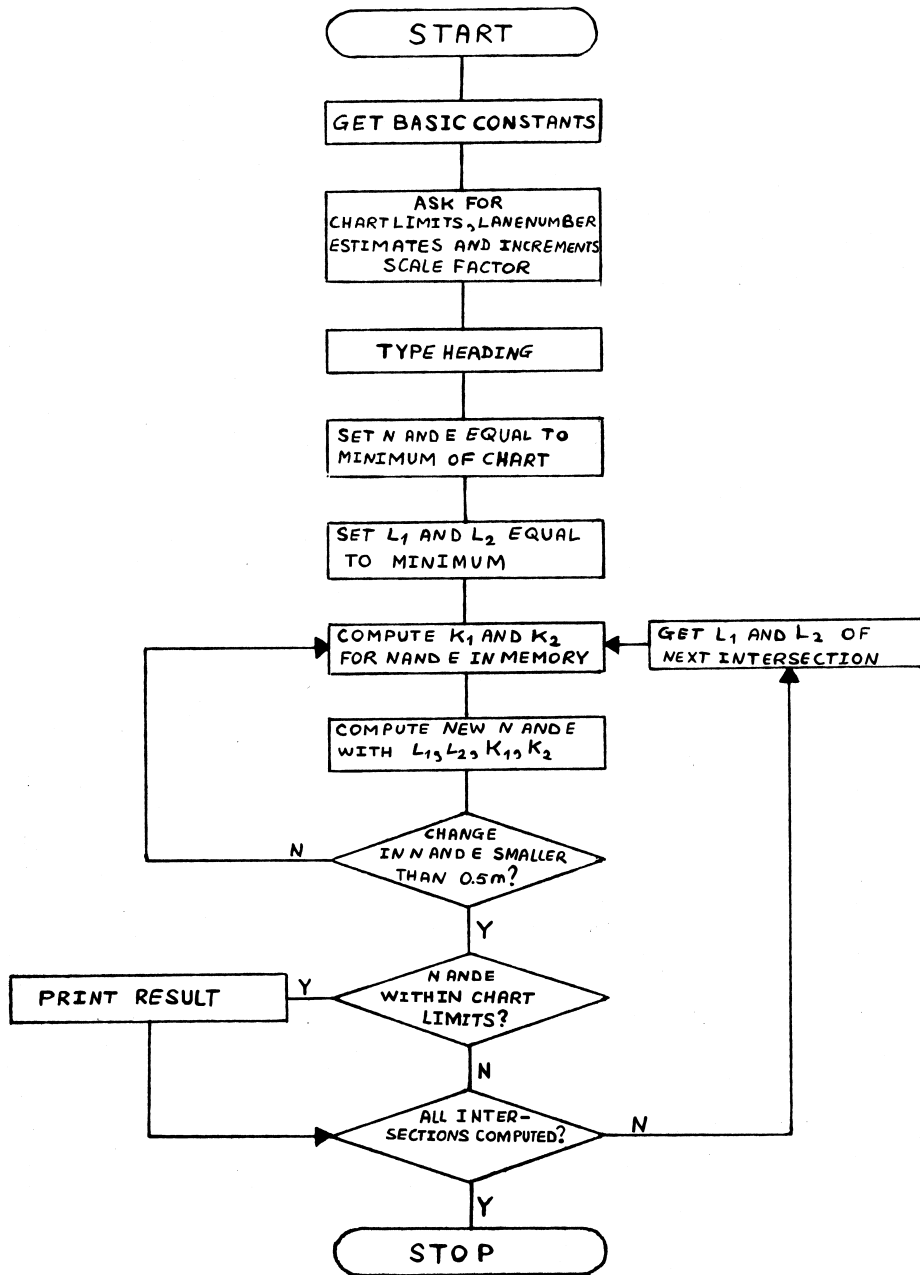


Figure 2. Flow chart of the programme.

Final remarks

1. As the northings and eastings are calculated in an iteration procedure a few words should be spent on the convergency of the method. As can be seen very easily the second derivatives in the series expansions (7) and (8) are inversely proportional to the third power of the distances P_1 , P_2 and P_M . So, the rest term in the series expansion, proportional to the second derivatives, is small compared with the first-order terms, and therefore the convergency is very fast.
2. A serious short-coming of the method is given by the problem of the scale factor. In our computer the memory is too small to store an extensive list of scale factors and we have to confine ourselves with an average of the scale factor for the area covered by the chart. Therefore, the same programme, although in itself very suitable, cannot be used for the construction of a lattice chart of a smaller scale. If a computer with a sufficiently large memory is available, a list of scale factors may be added and the method will turn out to be very advantageous for the computation of a hyperbolic lattice chart.
3. In the exceptional situation that the master station lies within the calibration chart, the programme may run into trouble. In this case some pairs of lanes intersect each other twice, but, in general, only one of these intersections will be found by the programme, which one being dependent on the final estimates of northing and easting. The result will be that the second set of intersections will not be found. However, by running the programme a second time with another 'first estimation' the remaining intersections may be determined.
4. In the case that any of the three stations falls within the area of the chart it should be considered that a division by zero may become possible. To avoid this the lower limits of both patterns must always be larger than zero and the higher limits must always be smaller than the lane numbers of the slave stations.

References

- [1] International Hydrographic Bureau, *Radio aids to maritime navigation and hydrography*, Monaco 1965.

- [2] Verstelle, J.Th.; '(Electronic) conversion of hyperbolic into rectangular coordinates', Supplement to the International Hydrographic Review **4** (1963) 39.
- [3] Digital Equipment Corporation, *Introduction to programming*, 1969.
- [4] Digital Equipment Corporation, *FOCAL programming manual*, Maynard 1968.

Note

In 1969 dr Peter Koeze was with Shell International Petroleum Company Ltd., London. He wrote this paper while on board a seismic survey vessel, MV 'Petrel', at the time of writing employed on various assignments in Far-Eastern waters. This digital version was composed in March 2007.

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Curriculum vitae: <http://www.pkoeze.nl/CurriculumVitae.pdf>

C-FOCAL., 1968

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01.10 S VC=0.299670E+9
01.20 S HZ=0.173500E+7
01.30 S B1=0.102944E+6
01.40 S N1=0.626174E+6
01.50 S E1=0.468054E+6
01.60 S B2=0.777326E+5
01.70 S N2=0.773226E+6
01.80 S E2=0.516120E+6
01.90 S NM=0.699399E+6
01.95 S EM=0.540353E+6
01.99 T "ADD 3,000,000 TO ALL PRINTED NORTHINGS." !

02.10 A "NORTHING LOW LIMIT "NL," HIGH ' ' "NH
02.20 A "EASTING LOW ' ' "EL," HIGH ' ' "EH
02.30 A "PATT I LOW ' ' "L1(1)," HIGH ' ' "L1(2)
02.40 A " INCREMENT "L1(3),"PATT II LOW LIMIT"L2(1)
02.50 A " HIGH ' ' "L2(2)," INCREMENT"L2(3)
02.60 A "SCALE FACTOR "SF

03.10 T !! " PATT I PATT II NORTH EAST "
03.20 S N=NL; S E=EL
03.30 F A=L1(1),L1(3),L1(2);F B=L2(1),L2(3),L2(2);D 3.5
03.40 QUIT
03.50 D 4;I (N-NL) 3.7;I (NH-N) 3.7;I (E-EL) 3.7;I (EH-E) 3.7,3.6,3.6
03.60 T %6.02,A," ",B," ",%7,N," ",E,!
03.70 RETURN
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(Continued on the next page.)

Table 1.

(Continuation from the previous page.)

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04.05 S P1=FSQT((N-N1)^2+(E-E1)^2)/SF
04.10 S P2=FSQT((N-N2)^2+(E-E2)^2)/SF
04.15 S PM=FSQT((N-NM)^2+(E-EM)^2)/SF
04.20 S K1=(B1+PM-P1)*HZ/VC
04.25 S K2=(B2+PM-P2)*HZ/VC
04.30 S D1=((N-NM)/PM-(N-N1)/P1)*HZ/VC/SF^2
04.35 S D2=((E-EM)/PM-(E-E1)/P1)*HZ/VC/SF^2
04.40 S D3=((N-NM)/PM-(N-N2)/P2)*HZ/VC/SF^2
04.45 S D4=((E-EM)/PM-(E-E2)/P2)*HZ/VC/SF^2
04.55 S DN=(D4*(A-K1)-D2*(B-K2))/(D1*D4-D2*D3)
04.60 S DE=(D3*(A-K1)-D1*(B-K2))/(D2*D3-D1*D4)
04.65 S N=N+DN;S E=E+DE
04.70 I (0.5-FABS(DN)) 4.05,4.05;I (0.5-FABS(DE)) 4.05,4.05
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Table 2. The programme for the calculation of lane intersections needed for the construction of a calibration chart.

VC(00)=+0.299670E+09	PROPAGATION VELOCITY
HZ(00)=+0.173500E+07	FREQUENCY
B1(00)=+0.102944E+06	BASELINE TO SLAVE 1
N1(00)=+0.626174E+06	NORTHING OF SLAVE 1
E1(00)=+0.468054E+06	EASTING OF SLAVE 1
B2(00)=+0.777326E+05	BASELINE OF SLAVE 2
N2(00)=+0.773226E+06	NORTHING OF SLAVE 2
E2(00)=+0.516120E+06	EASTING OF SLAVE 2
NM(00)=+0.699399E+06	NORTHING OF MASTER
EM(00)=+0.540353E+06	EASTING OF MASTER
NL(00)=+0.697300E+06	NORTHING LOW CHART LIMIT
NH(00)=+0.698000E+06	NORTHING HIGH CHART LIMIT
EL(00)=+0.534500E+06	EASTING LOW CHART LIMIT
EH(00)=+0.535200E+06	EASTING HIGH CHART LIMIT
L1(01)=+0.580000E+02	PATTERN I LOW LIMIT
L1(02)=+0.680000E+02	PATTERN I HIGH LIMIT
L1(03)=+0.100000E+01	PATTERN I INCREMENT
L2(01)=+0.290000E+02	PATTERN II LOW LIMIT
L2(02)=+0.370000E+02	PATTERN II HIGH LIMIT
L2(03)=+0.100000E+01	PATTERN II INCREMENT
SF(00)=+0.999620E+00	SCALE FACTOR
N@(00)=+0.697737E+06	NORTHING OF CURRENT POINT
E@(00)=+0.534253E+06	EASTING OF CURRENT POINT
A@(00)=+0.690000E+02	COUNTER OF PATTERN I
B@(00)=+0.380000E+02	COUNTER OF PATTERN II
P1(00)=+0.975233E+05	DISTANCE FROM POINT P TO SLAVE 1
P2(00)=+0.776664E+05	DISTANCE FROM POINT P TO SLAVE 2
PM(00)=+0.632441E+04	DISTANCE FROM POINT P TO MASTER
K1(00)=+0.680007E+02	APPROXIMATED LANE NUMBER PATT I
K2(00)=+0.369994E+02	APPROXIMATED LANE NUMBER PATT II
D1(00)=-0.577480E-02	DERIVATIVE PATT I TO NORTHING
D2(00)=-0.952112E-02	DERIVATIVE PATT I TO EASTING
D3(00)=+0.410860E-02	DERIVATIVE PATT II TO NORTHING
D4(00)=-0.694085E-02	DERIVATIVE PATT II TO EASTING
DN(00)=+0.132478E+00	CORRECTION FOR NORTHING
DE(00)=-0.182237E-02	CORRECTION FOR EASTING

Table 3. The symbol table of the programme. The comment is not contained in the programme but was typed in afterwards.